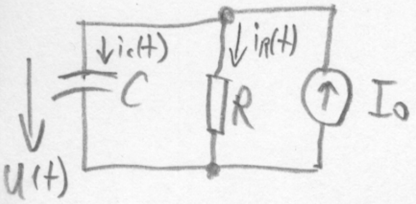
leichte Aufgabe



$$I_0 = i_C(t) + i_R(t) \quad , \quad u(t) = R \cdot i_R(t) = R \cdot \left(I_0 - i_C(t)\right)$$

$$i_C(t) = C \cdot \frac{du(t)}{dt}$$

$$i_C(t) = C \cdot \frac{d}{dt}\left(R \cdot I_0 - R \cdot i_C(t)\right) = \cancelto{0}{C\frac{d}{dt}(R \cdot I_0)} - RC\frac{di_C(t)}{dt}$$

$$i_C(t) = -RC\frac{di_C(t)}{dt} \leftarrow \text{GDE}$$

<u>Ansatz:</u> $\quad i_C(t) = A + B \cdot e^{\lambda t}$

Randbedingungen: $\quad i_C(t=0) = I_0 = A + B$

$$i_C(t \to \infty) = 0 = A \qquad (\lambda < 0)$$

$$\Rightarrow A = 0, \; B = I_0$$

$$\Rightarrow \quad i(t) = I_0 e^{\lambda t}$$

$$\frac{di(t)}{dt} = I_0 \lambda e^{\lambda t}$$ einsetzen in GDE $\Rightarrow$

$$I_0 e^{\lambda t} = -RC \cdot I_0 \lambda e^{\lambda t} \quad \Rightarrow \quad 1 = -RC \cdot \lambda$$

$$\Rightarrow \boxed{\lambda = -\frac{1}{RC}}$$

$$\Rightarrow \boxed{i(t) = I_0 \cdot e^{-\frac{t}{RC}}}$$ und: $u(t) = R \cdot \left(I_0 - I_0 \cdot e^{-\frac{t}{RC}}\right)$

wichtig $\rightarrow$ $$\boxed{u(t) = R \cdot I_0 \cdot \left(1 - e^{-\frac{t}{RC}}\right)}$$

Bestimmen von R:

$$u(t_1) \stackrel{!}{=} U_1 \quad , \left(t_1 = 60\,\mathrm{ms},\ U_1 = 5\,\mathrm{V},\ C = 47\,\mathrm{nF},\ I_0 = 5\,\mu\mathrm{A} \right)$$

$$\Downarrow$$

$$U_1 = R \cdot I_0 \cdot \left(1 - e^{-\frac{t_1}{RC}}\right) = R \cdot I_0 - R \cdot I_0 \cdot e^{-\frac{t_1}{RC}}$$

$$R \cdot I_0 \cdot e^{-\frac{t_1}{R \cdot C}} = R \cdot I_0 - U_1$$

$$e^{-\frac{t_1}{R \cdot C}} = 1 - \frac{U_1}{R \cdot I_0}$$

Substitution $x := \frac{1}{R} \Rightarrow e^{-\frac{t_1}{C} \cdot x} = 1 - \frac{U_1}{I_0} \cdot x$

$$\alpha := \frac{t_1}{C} \quad , \quad \beta := -\frac{U_1}{I_0}$$

$$e^{-\alpha x} = 1 + \beta \cdot x$$

$$\gamma := \frac{-1}{\beta} = + \frac{I_0}{U_1}$$

$$\Rightarrow e^{-\alpha x} = -\gamma \cdot \beta + \beta \cdot x = \beta \cdot (x - \gamma)$$

(Wikipedia)

$$\Rightarrow \quad x = \gamma + \frac{1}{\alpha} \cdot W_0\left(\frac{\alpha}{\beta} e^{-\alpha \cdot \gamma} \right) = \frac{1}{R}$$

$$\Rightarrow R = \frac{1}{\frac{I_0}{U_1} + \frac{C}{t_1} \cdot W_0\left(-\frac{t_1}{C} \cdot \frac{I_0}{U_1} \cdot e^{-\frac{t_1}{C} \cdot \frac{I_0}{U_1}} \right)}$$

Lambertsche W-Funktion!