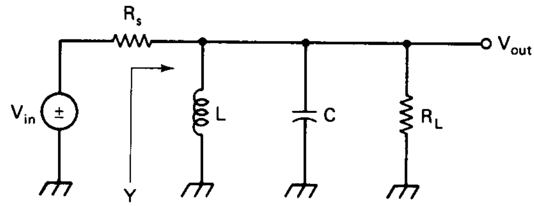

Figure 2.15 A parallel tuned circuit, doubly terminated.

Eq. 2.3-3. Some manipulation then shows that the Q equals half that of the series tuned circuit. However, the series tuned circuit studied contained only one resistor while the parallel one of Fig. 2.15 has two. The circuit Q is thus given by $Q_L = R_p/\omega L$ where R_p is now the parallel combination of the source and load resistors. Note that the same form of transfer function describes both the series and the parallel tuned circuit.

Assume now that the components in the filter of Fig. 2.15 are not ideal, but have a loss described by measurable Q_u values. The equivalent parallel resistance associated with the inductor is $R_{pL} = Q_{uL}\omega L$ while the loss in the capacitor is $R_{pC} = Q_{uC}/\omega C$. But $L = 1/\omega C$. The equivalent loss resistance is the parallel combination of the two. The final result is the equivalent Q_u

$$Q_{\text{net}} = (Q_{uL}^{-1} + Q_{uC}^{-1})^{-1} \tag{2.6-9}$$

This equation is quite general. When a number of lossy reactances are connected in parallel or in series, the equivalent Q is obtained in the same way that the equivalent resistance of parallel resistors is obtained.

Assume that the Q_u value of the capacitor is arbitrarily high. The Q_u of the tuned circuit is then that of the inductor alone, resulting in $R_p = Q_u\omega L$. The effects of the reactive components disappear at resonance, leaving the equivalent circuit shown in Fig. 2.16. The transducer gain of this network will be evaluated. We assume that $R_s = R_L = R$ and the voltage generator has an amplitude of 2 V. The output voltage is written from inspection. Algebraic reduction produces the result

$$V_{\text{out}} = \frac{2\left(\dfrac{R_pR}{R+R_p}\right)}{R + \dfrac{R_pR}{R+R_p}} = \frac{1}{1 + R/2R_p} \tag{2.6-10}$$

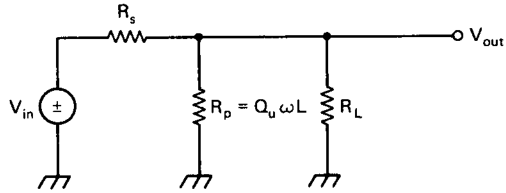

Figure 2.16 The equivalent of the parallel tuned circuit at resonance. R_p is the resistance representing the losses in the resonator.

The unloaded Q is $Q_u = R_p/\omega L$ while the loaded Q is $Q_L = R_e/\omega L$ where R_e is the equivalent resistance loading the inductor. R_e is the parallel combination of the load, the source, and R_p. Hence

$$R_e = \frac{R_p R}{2R_p + R} \tag{2.6-11}$$

for the case of equal load and source resistances.

Consider the ratio of Q_L/Q_u

$$\frac{Q_L}{Q_u} = \frac{R_e}{R_p} = \frac{R}{2R_p + R} \tag{2.6-12}$$

This simplifies to the relationship

$$\frac{R}{2R_p} = \frac{Q_L}{Q_u - 1} \tag{2.6-13}$$

which is then inserted into Eq. 2.6-10 to produce

$$V_{\text{out}} = \left(1 - \frac{Q_L}{Q_u}\right) \tag{2.6-14}$$

Because the source and load resistances are equal, the insertion loss of the filter of Fig. 2.15 is the negative of the transducer gain, or

$$\text{Insertion loss (IL)} = -20 \log (1 - Q_L/Q_u) \text{ db} \tag{2.6-15}$$

This filter is a doubly terminated single resonator.

The bandwidth of the single resonator, terminated or not, is analytically related to the Q. Recalling that the bandwidth is defined as the frequency where the output power is down by half, or 3 dB, bandwidth is related to center frequency and Q by

$$Q = f_0/\text{BW} \tag{2.6-16}$$

where F_0 and BW are both measured in the same units.

This is well illustrated by an example. Assume a resonator has a center frequency of 100 MHz and $Q_u = 400$. The unloaded bandwidth is $100/400 = 0.25$ MHz (Q is a dimensionless number). A bandwidth of 1 MHz is measured if the resonator is then placed between an equal source and load. The loaded Q is then $Q_L = 100/1 = 100$. The insertion loss from Eq. 2.6-15 is 2.5 dB.

Examination of the equations reveals a method for measuring the Q of a resonator. Note that as the loaded Q of a filter approaches the unloaded value, the insertion loss becomes very large. If a resonator is terminated equally by both the generator and load and the values are adjusted so that the insertion loss is very large, the measured 3-dB bandwidth will produce a loaded Q according to Eq. 2.6-16. Measuring the insertion loss will allow calculation of Q_L/Q_u. The loaded Q is very close to Q_u if the insertion loss is high enough, typically 30 to 40 dB.

Q has been related to a second order network in our discussion. That is, Q is a parameter of a network containing two reactive elements, described by a transfer function of second order. Sometimes a parameter Q appears in design equations for third or even fourth order networks. The meaning for this Q is only loosely related to the Q parameter we have been discussing. The actual energy storage in the network and the related bandwidth properties are sometimes completely unrelated to Q.

2.7 THE ALL-POLE LOW PASS FILTER

Previous sections have presented background information and analysis methods. Now, the problem of filter design is finally approached. The type of filter considered in this section is shown in Fig. 2.17. It contains only series inductors and shunt capacitors. The filter is a ladder configuration and may be analyzed using the ladder method presented earlier.

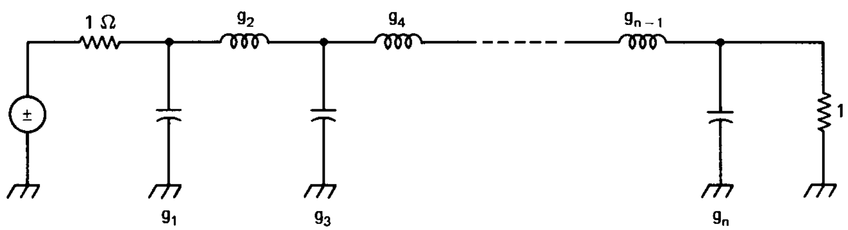

Figure 2.17 An *n*th order low pass filter. The text has equations for evaluating the component values and the location of the poles.

We find that the transfer function, $H(s)$, of the filter of Fig. 2.17 contains no finite zeros. The transfer function is the reciprocal of a simple polynomial in the variable s. As such, the filter is described completely by factoring of the denominator polynomial to extract the location of the complex poles. This filter, an *all-pole* type, represents many of the practical low pass filters used in routine rf design.

Most design work with all filters is done with low pass prototypes like that of Fig. 2.17 where the source and load resistance are 1 Ω and the cutoff frequency is 1 rad. This has a number of consequences. First, analysis is simplified. Second, the normalized filters are in a form that may be easily scaled to other terminations and cutoff frequencies. The final rationale is somewhat less obvious, though. At a frequency of 1 rad s^{-1}, a 1-H inductor and a 1-F capacitor have the same immittance, 1 Ω or 1 Siemen (S). This duality allows us to treat them more easily than we could if another frequency of normalization were chosen. One consequence is that one form of filter may be transformed into another with no change in numerical values.